Chaotic Systems as 3D Height Maps for Sound Synthesis

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ABSTRACT

This paper introduces a method for generating dynamic audiovisual landscapes by applying a series of discrete-time chaotic systems to wave terrain synthesis (WTS). Chaotic oscillators are known for their sensitive dependence on initial conditions and their tendency to produce complex behaviors—often at the risk of overloading audio engines. This research proposes using these chaotic systems for terrain generation by plotting them as 3D height maps, avoiding the typical pitfalls of instability rather than direct sound production which is commonly the focus of music and chaos related research. This approach also preserves the timbral richness of chaotic behavior while offering improved harmonic synthesis when periodic waveforms are used to read from the terrain. Beyond sound synthesis for electronic music, chaotic height maps offer opportunities for visual representation, making the system both an auditory and visual instrument. This paper positions chaotic wave terrain synthesis within the broader context of interdisciplinary audiovisual research areas, such as sound design and data visualization, and is part of an ongoing project (IRESAP) concerned with strategies incorporating current music information retrieval techniques and audiovisual artistic practices.

Author Keywords

Nonlinear Dynamics, Wave Terrain Synthesis (WTS), Sound Design, Data Visualization, Digital Signal Processing, Audiovisual Instruments

1. CHAOTIC SYSTEMS

Chaotic systems are dynamic systems described by nonlinear ordinary differential equations, where nonlinearity is essential for chaotic behavior. Beyond their theoretical interest, chaotic systems have practical applications across physics, biology, engineering, economics, and music [16]. One example is the Thomas System, which demonstrates how certain parameter values create complex patterns known as strange attractors, or a set of states toward which a system tends to evolve (Figure 1). While these systems may sometimes appear unpredictable, they are not random; they follow deterministic rules and are highly sensitive to their initial conditions. These key characteristics set them apart from linear or predictable systems; this project focuses particularly on the nonlinearity, deterministic nature, sensitivity to initial conditions, and strange attractors of these systems [5].

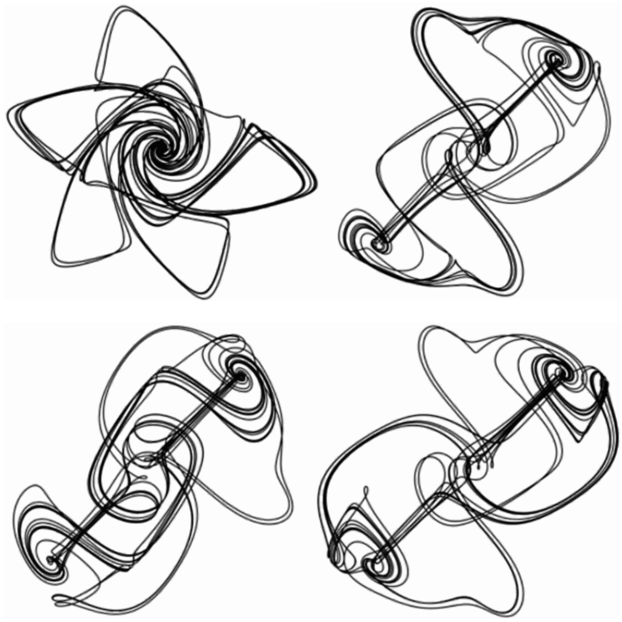

Figure 1: The Thomas System exhibiting states that demonstrate strange attractors.

Computer music researchers began exploring discrete-time chaotic systems based on differential equations in the late 1980s and early 1990s, leading to various applications, from control-rate parameter manipulation [20] to audio-sampling rate oscillation for complex timbres [15]. Chaotic systems have also been mapped and quantized to produce MIDI note sequences [9]. However, these systems are known for being unstable and can easily overload audio engines, requiring re-instantiation to proceed with sound production. Several strategies to address this challenge have been proposed, including connecting chaotic systems to digital waveguides [19], chaotic sound spatialization [17], or using bounding and clipping functions to maintain stability [14]. Recently, researchers have focused on using chaotic equations to influence audio inputs rather than directly synthesizing sound [8]. Due to these inherent issues, chaotic systems are often seen as having "razor-thin" musical applications [1].

Chaotic oscillators also struggle to produce harmonic tones due to their aperiodicity, which prevent them from generating cycles that decompose into harmonic components [6].

2nd Audiovisual Symposium 2024 December 6, 2024, Dalarna Audiovisual Academy (DAVA), Dalarna University, Falun, Sweden

Unlike traditional oscillators that produce stable, periodic waveforms, chaotic oscillators produce signals that are inherently unstable. This lack of periodicity means that the typical harmonic series—an orderly stack of integer multiples of a fundamental frequency—is disrupted, making it nearly impossible for chaotic oscillators to generate the kind of harmonic structure that traditional tonal music relies upon. Instead, chaotic oscillators are better suited for noise-based, non-linear synthesis or non-western music practices [7]. As a result, chaotic oscillators excel in experimental sound design, where rich, evolving textures take precedence over harmonic clarity and predictability.

2. WAVE TERRAIN SYNTHESIS

Wave Terrain Synthesis (WTS) is a sound synthesis technique that traces a trajectory across a three-dimensional surface, or "terrain," using the resulting height values to generate audio. In essence, WTS uses a 3D surface as a lookup table: coordinates on the terrain serve as inputs, and the height at each point represents amplitude. Despite its broad scope and limited research, WTS is often situated within the realms of wavetable synthesis [13], FM synthesis [11], waveshaping and distortion synthesis [4], and wavetable cross-multiplication (e.g., ring modulation) [2].

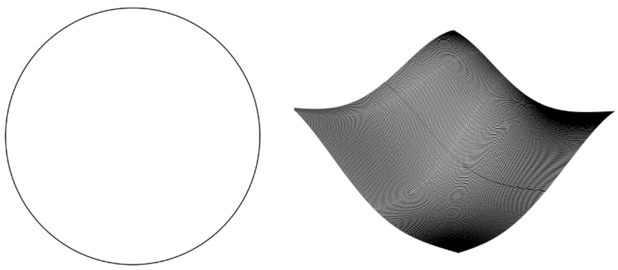

Figure 2: A sinusoidal oscillator that forms a circle in (x,y)-space (left) and a 3D height map in (x,y,z)-space (right).

Consider the periodic sinusoidal oscillator in Figure 2, which traces a circular series of points in (x,y)-space, with the x- and y-coordinates generating independent sinusoidal outputs. In (x,y,z)-space, where the z-coordinate is the sum of the x- and y-amplitudes, a 2D audio signal is transformed into a 3D height map. Reading from a height map is analogous to rolling a ball over a hilly landscape [12]. Both the terrain and the path by which the ball moves over the landscape are defined independently of one another. However, both structures are mutually dependent in finding an outcome, and any changes that occur in either system affect the resulting waveform [10].

Height maps are often generated using basis functions, such as the Wave Terrain Synthesis package for Max by Timo Hoogland at IRCAM, which supports polar and Cartesian lookup.[1] Few software tools exist for wave terrain synthesis, leading many to develop their own. Max includes the linear terrain~ object in the PeRColate library [18], and 2d.wave~ offers a two-variable wavetable for scanning trajectories [21]. Additionally, the MacOS application DrawJong 2.0 can visualize and sonify a wide range of chaotic systems [9]. Hardware applications are rarer, but one example is Samuel Carswell's Eurorack WTS module, which enables voltage-controlled trajectory specification and CV output [3].

[1]https://forum.ircam.fr/projects/detail/wave-terrain-synthesis/

3. IMPLEMENTATION

This section presents a series of six (6) chaotic systems represented in both (x,y) and (x,y,z)-space. These system were chosen based on their visual appeal from an online repository for chaotic systems.[2] In cases where the systems produce an (x,y,z) output—such as the Hindmarsh-Rose Attractor—only the (x,y)-coordinates are taken and used to generate terrains. For each system, a brief description and its equation are introduced. The image of each system includes its 2D and 3D height map representations, along with the initial state for the variables defined in each equation. Each of these systems were implemented in gen~, an embedded environment in Max that processes audio at the audio sampling rate rather than the vector rate.[3]

Visualizing a height map is accomplished using Jitter matrices in Max, which enables video processing and graphics rendering.[4] Each map captures a static "snapshot" of n samples from the chaotic system, requiring the oscillators to only function temporarily. This allows for saving and recalling presets, reducing instability associated with chaotic oscillators. Once created, each dimension of the map can be independently scaled or otherwise modified without relying on the oscillators. Two maps can also be interpolated to generate hybrid timbres. Although maps can dynamically change as the system oscillates, doing so risks audio engine overload. When the lookup method for a terrain is periodic, harmonicity is preserved, allowing the system's rich, nonlinear dynamics to be heard in the resulting overtones.

3.1 Duffing System

The Duffing system (Figure 3) is a type of dynamical system that models non-linear oscillatory behavior, often used to describe forced damped oscillators with a non-linear stiffness component. Its behavior can transition from periodic to chaotic, depending on parameters like the strength of the forcing and the level of damping.

$$\ddot{x} + \delta\dot{x} + \alpha x + \beta x^3 = \gamma \cos(\omega t)$$

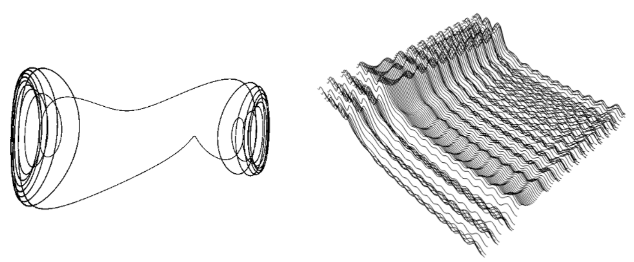

Figure 3: A Duffing Attractor (left) and 3D height map (right) using coefficients $\delta = 0.5$, $\alpha = 1$, $\beta = 0.1$, $\gamma = 1000$, $\omega = 123$, $t = 0.001$, with starting variable $x = 0$.

3.2 Hindmarsh-Rose System

The Hindmarsh-Rose system (Figure 4) is a mathematical model used to describe the electrical activity of neurons, particularly their spiking and bursting behaviors. It's a simplified version of more complex neuron models like the Hodgkin-Huxley model, but it's still rich enough to capture the chaotic dynamics of neuronal firing.

[2]http://www.3d-meier.de/tut19/Seite0.html
[3]https://docs.cycling74.com/max8/refpages/gen
[4]https://cycling74.com/products/jitter

$$
\begin{aligned}
\dot{x} &= y - ax^3 + bx^2 - z + I \\
\dot{y} &= c - dx^2 - y \\
\dot{z} &= r\left(s(x - x_0) - z\right)
\end{aligned}
$$

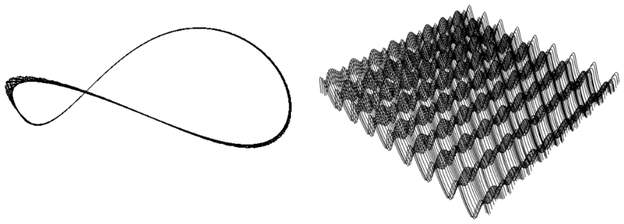

Figure 4: A Hindmarsh-Rose Attractor (left) and 3D height map (right) using coefficients a = 0.49, b = 1, c = 0.0322, d = 1, s = 1, v = 0.8, u = 0.03, with starting variables (x,y,z) = 0.1.

3.3 Lorenz System

The Lorenz system (Figure 5) is a set of three differential equations that describe chaotic behavior in a simplified model of atmospheric convection. Developed by Edward Lorenz in the 1960s, this system is one of the most famous examples of chaos theory, illustrating how small changes in initial conditions can lead to vastly different outcomes.

$$
\begin{aligned}
\dot{x} &= \sigma(y - x) \\
\dot{y} &= x(\rho - z) - y \\
\dot{z} &= xy - \beta z
\end{aligned}
$$

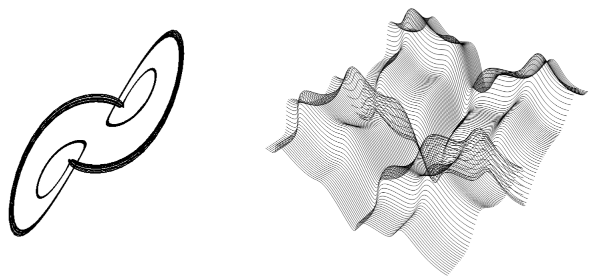

Figure 5: A Lorenz Attractor oscillator (left) and 3D height map (right) using coefficients $\sigma = 36$, $\rho = 49$, $\beta = 2.666667$, x = 0.1, with starting variables $(y, z) = 0$.

3.4 Peter de Jong System

The Peter de Jong systems (Figure 6) is a chaotic system that generates intricate, often nature-like patterns through simple recursive equations. Its adjustable parameters produce visually striking results, making it popular in generative art. Artists and scientists alike use it for visualizing complex systems and creating audio-visual works with dynamic, chaotic mappings.

$$
\begin{aligned}
x_{n+1} &= \sin(ay_n) + c\cos(ax_n) \\
y_{n+1} &= \sin(bx_n) + d\cos(by_n)
\end{aligned}
$$

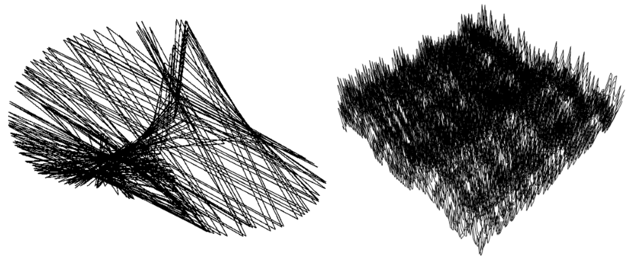

Figure 6: A Peter de Jong Attractor oscillator (left) and 3D height map (right) using coefficients a = 1.641, b = 1.902, c = 0.316, d = 1.512 with starting variables (x,y) = 1.

3.5 Thomas System

The Thomas system (Figure 7) is a chaotic attractor defined by a set of differential equations that generate swirling, cloud-like patterns. Its behavior results in complex structures that are popular in visualizations of chaotic dynamics. Artists and scientists use the Thomas system to explore chaotic motion and visualize turbulent flows.

$$
\begin{aligned}
\dot{x} &= -bx + \sin(y), \\
\dot{y} &= -by + \sin(z), \\
\dot{z} &= -bz + \sin(x),
\end{aligned}
$$

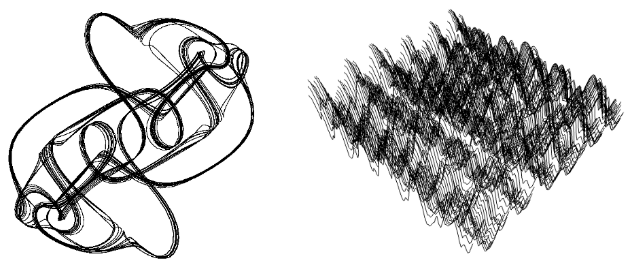

Figure 7: A Thomas Attractor oscillator (left) and 3D height map (right) using coefficients b = 0.19, x = 0.44, y = 0.21, with starting variables z = -0.56.

3.6 Tinkerbell System

The Tinkerbell system (Figure 8) is a simple chaotic map, named for the whimsical patterns it can produce, resembling a butterfly or fairy-like figure. It is a two-dimensional discrete dynamical system that exhibits chaotic behavior under certain conditions.

$$
\begin{aligned}
x_{n+1} &= x_n^2 - y_n^2 + ax_n + by_n \\
y_{n+1} &= 2x_ny_n + cx_n^2 + dy_n^2
\end{aligned}
$$

4. CONCLUSIONS AND FUTURE WORK

This paper has introduced the use of chaotic systems as 3D height maps in the context of wave terrain synthesis, providing a sound synthesis method that avoids the instability of chaotic oscillators in direct audio production. By using chaotic systems to shape terrains instead of generating sound directly, the complexity of chaotic behavior is preserved while offering improved harmonic synthesis. This technique not only expands possibilities for sound design but also enhances audiovisual performance, deepening the connection between visual and auditory elements. Future

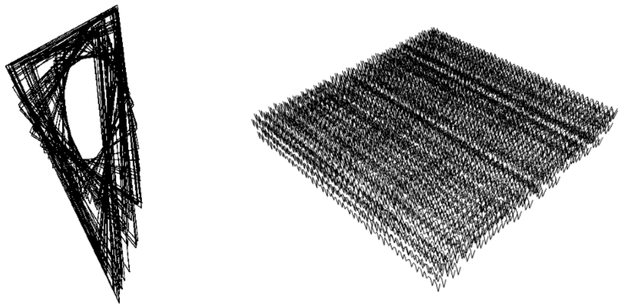

Figure 8: A Tinkerbell Attractor oscillator (left) and 3D height map (right) using coefficients a = -0.3, b = -0.6, c = 2, d = -0.27 with starting variables x = -0.72 and y = -0.64.

work will explore additional chaotic systems and develop open-source tools to make this synthesis method more accessible to musicians and multimedia artists, expanding creative potential across sound and visual art practices and enriching interdisciplinary research.

5. ACKNOWLEDGMENTS

Information Retrieval in Embedded Systems for Audiovisual Artistic Processes (IRESAP) is supported by The Knowledge Foundation (KKS).

6. REFERENCES

[1] E. Berdahl, E. Sheffield, A. Pfalz, and A. T. Marasco. Widening the razor-thin edge of chaos into a musical highway: Connecting chaotic maps to digital waveguides. In *NIME*, pages 390–393, 2018.

[2] A. Borgonovo and G. Haus. Sound synthesis by means of two-variable functions: experimental criteria and results. *Computer Music Journal*, 10(3):57–71, 1986.

[3] S. Carswell. *Wave Terrain Hardware for Modular Synthesis*. PhD thesis, Open Access Te Herenga Waka-Victoria University of Wellington, 2023.

[4] J. Comajuncosas. Wave terrain synthesis with csound. *The csound book: perspectives in software synthesis, sound design, signal processing, and programming. MIT Press, Cambridge, MA*, 2000.

[5] W. Ditto and T. Munakata. Principles and applications of chaotic systems. *Communications of the ACM*, 38(11):96–102, 1995.

[6] N. Fletcher. Inharmonicity, nonlinearity and music. *The Physicist*, 37(5):171–175, 2000.

[7] N. H. Fletcher et al. *Nonlinear dynamics and chaos in musical instruments*. IOS press, Amsterdam, 1993.

[8] M. Havryliv. Composing for improvisation with chaotic oscillators. 2010.

[9] M. Hetrick. Drawjong 2.0. 2011.

[10] S. G. James. Developing a flexible and expressive realtime polyphonic wave terrain synthesis instrument based on a visual and multidimensional methodology. 2005.

[11] Y. Mitsuhashi. Audio signal synthesis by functions of two variables. *Journal of the Audio Engineering Society*, 30(10):701–706, 1982.

[12] J. C. Nelson. Understanding and using csound's gen routines. *The Csound Book*, 2000.

[13] C. Roads. *Computer Music Tutorial*, volume 170. MIT Press, 1996.

[14] S. Sastry. *Nonlinear systems: analysis, stability, and control*, volume 10. Springer Science & Business Media, 2013.

[15] D. Slater. Chaotic sound synthesis. *Computer Music Journal*, 22(2):12–19, 1998.

[16] D. Solev, P. Janjic, and L. Kocarev. Introduction to chaos. In *Chaos-Based Cryptography: Theory, Algorithms and Applications*, pages 1–25. Springer, 2011.

[17] E. Soria and R. Morales-Manzanares. Multidimensional sound spatialization by means of chaotic dynamical systems. In *NIME*, volume 13, pages 79–83, 2013.

[18] D. Trueman and R. L. DuBois. A collection of synthesis, signal processing, and video objects (with source-code toolkit) for max/msp/nato. 2001.

[19] L. P. Viator. *Finding Music in Chaos: Designing and Composing with Virtual Instruments Inspired by Chaotic Equations*. Louisiana State University and Agricultural & Mechanical College, 2020.

[20] S. Zabetian. *Extensions to Dynamic Wave Terrain Synthesis for Multidimensional Polyphonic Expression*. School of Information and Communication Technology, Aalborg University, 2018.

[21] D. Zicarelli. Writing external objects for max 4.0 and msp 2.0. *San Francisco: Cycling*, 74, 2001.